Module 6 Particles & Medical Physics

Module 6: Particles and medical physics

In this module, learners will learn about capacitors, electric field, electromagnetism, nuclear physics, particle physics and medical imaging.

Module 6 Particles & Medical Physics

Unit 1 Capacitors

6.1 Capacitors

This section introduces the basic properties of capacitors and how they are used in electrical circuits. The use of capacitors as a source of electrical energy is then developed. This section introduces the mathematics of exponential decay, which is also required for the decay of radioactive nuclei in **6.4**.

This section provides knowledge and understanding of capacitors and exponential decay.

Experimental work provides an excellent way to understand the behaviour of capacitors in electrical circuits and the management of safety and risks when using power supplies (HSW4). There are many opportunities for learners to use spreadsheets in the analysis and presentation of data (HSW3). The varied uses of capacitors give the opportunity for the consideration of their use in many practical applications (HSW2, 5, 6, 9)

Module 5 – Newtonian world and astrophysics

5.1 Thermal physics
5.2 Circular motion
5.3 Oscillations
5.4 Gravitational fields
5.5 Astrophysics and cosmology

Module 6 – Particles and medical physics

You are here! →

6.1 Capacitors
6.2 Electric fields
6.3 Electromagnetism
6.4 Nuclear and particle physics
6.5 Medical imaging

6.1 Capacitors

- 6.1.1 Capacitors
- 6.1.2 Energy
- 6.1.3 Charging & Discharging Capacitors

6.1.1 Capacitors

6.1.1 Capacitors

Learning outcomes

Learners should be able to demonstrate and apply their knowledge and understanding of:

(a) capacitance; $C = \frac{Q}{V}$; the unit farad

(b) charging and discharging of a capacitor or capacitor plates with reference to the flow of electrons

(c) total capacitance of two or more capacitors in series; $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + ...$

(d) total capacitance of two or more capacitors in parallel; $C = C_1 + C_2 + ...$

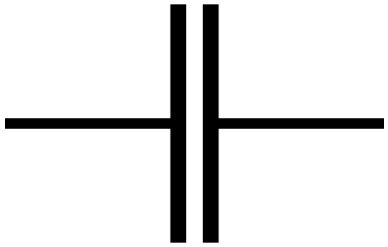
(e) **(i)** analysis of circuits containing capacitors, including resistors

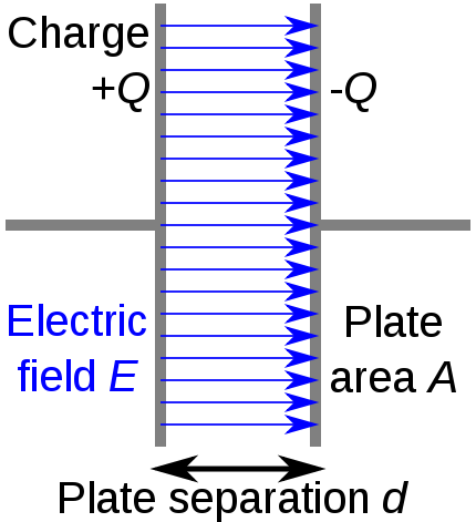
(ii) techniques and procedures used to investigate capacitors in both series and parallel combinations using ammeters and voltmeters.

What is a capacitor?

Capacitors

- Electrical components in which charges are kept apart on two metallic plates separated by an insulator called a dielectric.

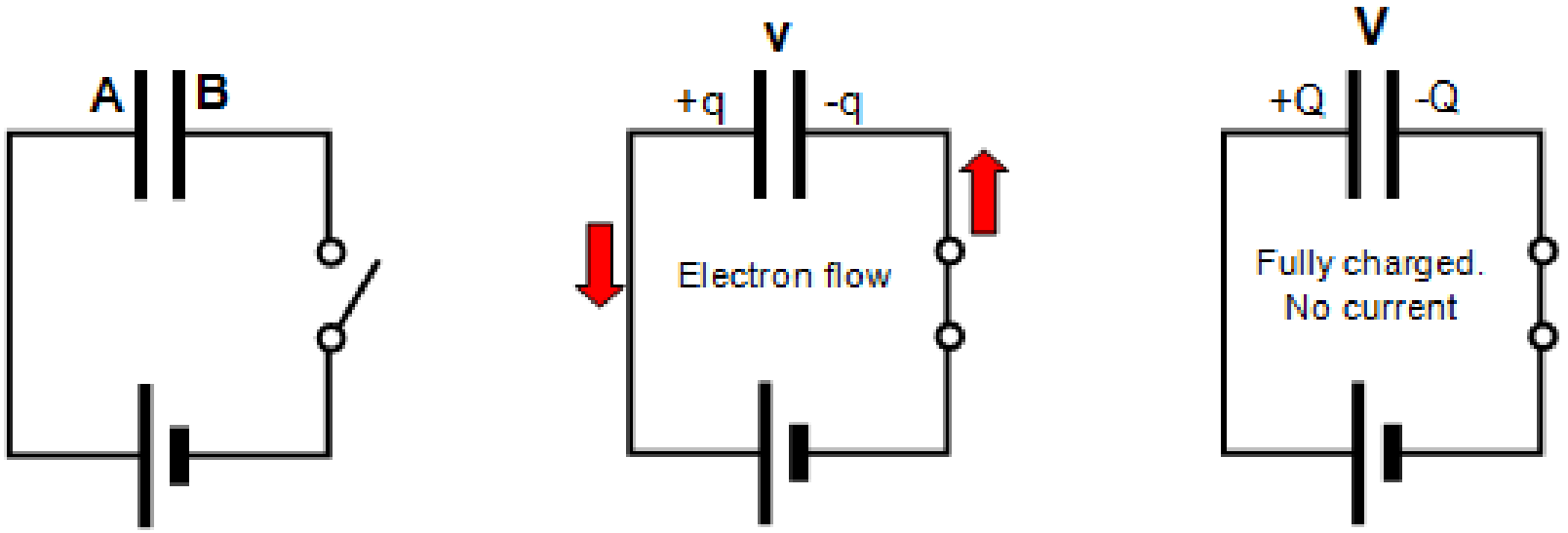




Charge Storage

- Once connected, electrons flow from the cell to plate B and from plate A to the cell.
- Plate B gains electrons so becomes negative while plate A loses electrons to become positive.

- The current falls to zero once the PD across the plates equals the EMF across the cell.
- The capacitor is now fully charged, with –Q and +Q.



Capacitance

- The amount of charge a capacitor can store per unit Potential Difference.

$$C = \frac{Q}{V}$$

The unit of capacitance is the farad.

$1F = 1CV^{-1}$

Where:

C = Capacitance, in farads (F)

Q = Charge stored, in coulombs (C)

V = Potential difference, in volts (v)

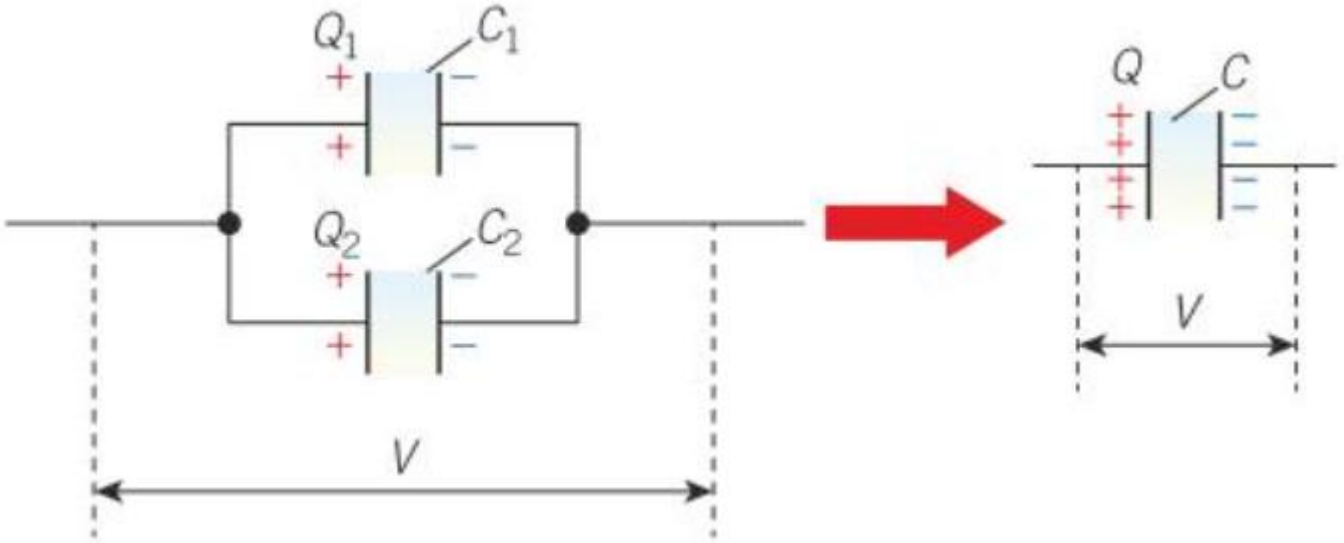
Rearranging…

$$C = \frac{Q}{V} \qquad \text{to} \qquad Q = VC$$

- The more charge stored on the plates, the greater the PD across them.
- Charge is therefore proportional to PD.

How do we add capacitance in series or parallel circuits?

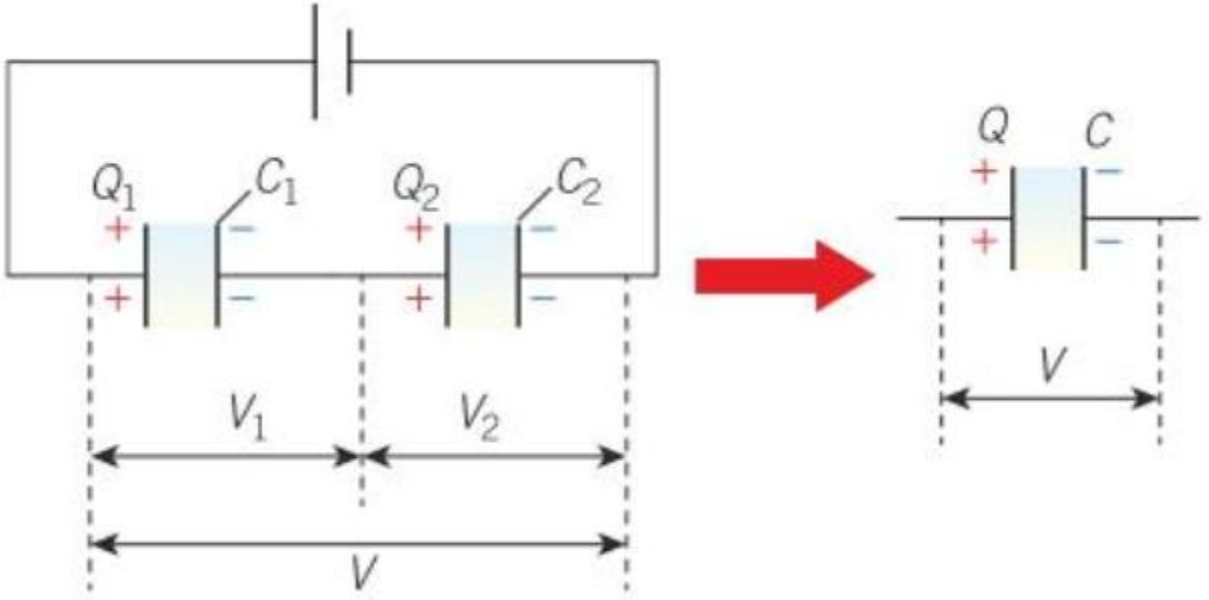
Connecting Capacitors – In Parallel



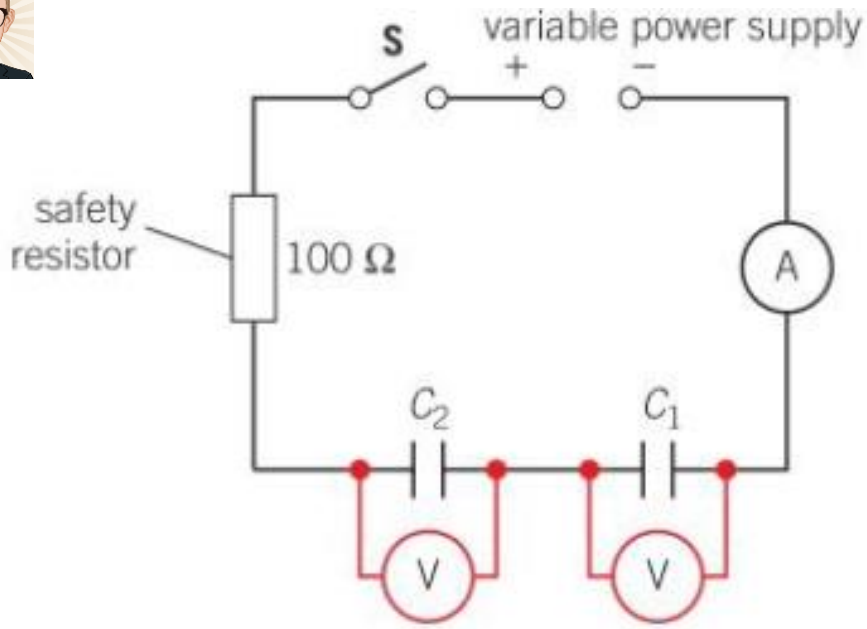
- The PD across the capacitors is the same.
- Charge is conserved, $Q_{total} = Q_1 + Q_2 + \ldots$

- So total capacitance is the sum of individual capacitances, $C_{total} = C_1 + C_2 + \ldots$

Connecting Capacitors – In Series



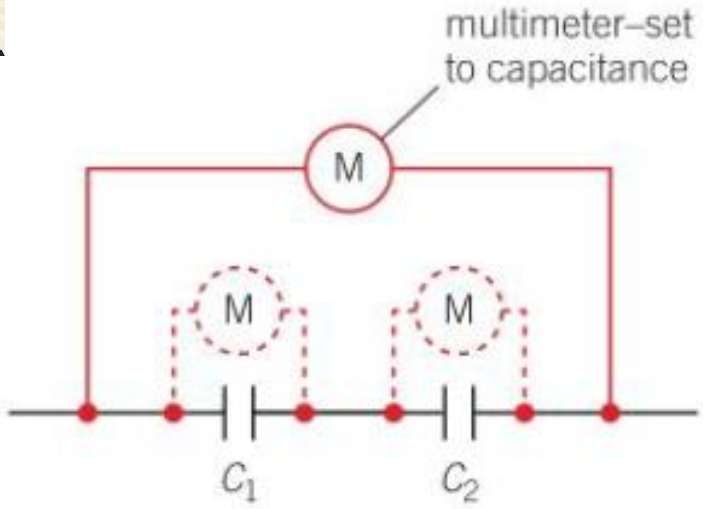
- All capacitors in series store the same charge, even if they have different capacitances.
  - Since the cell is connected to the left plate of C_1 and the right plate of C_2.
  - The middle plates are not connected to the cell but do become charged with the same magnitude, Q.
- The total PD across the capacitors is $V = V_1 + V_2 + \ldots$
- So total capacitance is given by: $\frac{1}{C_{Tot}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots$



Practical

- Set up the circuit shown.
- Watch the ammeter briefly show a current as the capacitors charge.
  - Electrons only flow while charging.
- Measure & record PD across each capacitor.
- Calculate charge stored on each capacitor. $Q = VC$

- Show that charges, Q_1 and Q_2 are equal.



Practical

- Set up a variety of combinations of capacitors in series and in parallel.
- Use a multimeter set to capacitance to measure the total capacitance of the whole set up.
- Compare to the calculated capacitance using the addition rules:
- Record results in a table.

Rules

Adding capacitors in series:

$$\frac{1}{C_{Total}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots$$

Adding capacitors in parallel:

$C_{Total} = C_1 + C_2 + \ldots$

Proving the addition rule for series circuits

According to Kirchhoff's second law

$$V = V_1 + V_2$$

The charge Q stored by each capacitor is the same. Once again, you can use the equation $Q = VC$ for individual components or the entire circuit. Therefore

$$\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

The charge Q cancels out, leaving the equation for total capacitance

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}.$$

Proving the addition rule for parallel circuits

The total charge stored Q is equal to the sum of the individual charges, that is

$$Q = Q_1 + Q_2$$

The p.d. V across each capacitor is the same because they are connected in parallel. You can use the equation $Q = VC$ for individual components or the entire circuit. Therefore

$$VC = VC_1 + VC_2$$

The p.d. V cancels out leaving the equation for total capacitance, $C = C_1 + C_2$.

6.1.1 Capacitors (review)

6.1.1 Capacitors

Learning outcomes

Learners should be able to demonstrate and apply their knowledge and understanding of:

(a) capacitance; $C = \frac{Q}{V}$; the unit farad

(b) charging and discharging of a capacitor or capacitor plates with reference to the flow of electrons

(c) total capacitance of two or more capacitors in series; $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + ...$

(d) total capacitance of two or more capacitors in parallel; $C = C_1 + C_2 + ...$

(e) **(i)** analysis of circuits containing capacitors, including resistors

(ii) techniques and procedures used to investigate capacitors in both series and parallel combinations using ammeters and voltmeters.

6.1.2 Energy

6.1.2 Energy

Learning outcomes

Learners should be able to demonstrate and apply their knowledge and understanding of:

(a) p.d.–charge graph for a capacitor; energy stored is area under graph

(b) energy stored by capacitor;

$$W = \frac{1}{2}QV,\ W = \frac{1}{2}\frac{Q^2}{C} \text{ and } W = \frac{1}{2}V^2C$$

(c) uses of capacitors as storage of energy.

How do we represent energy stored by capacitors?

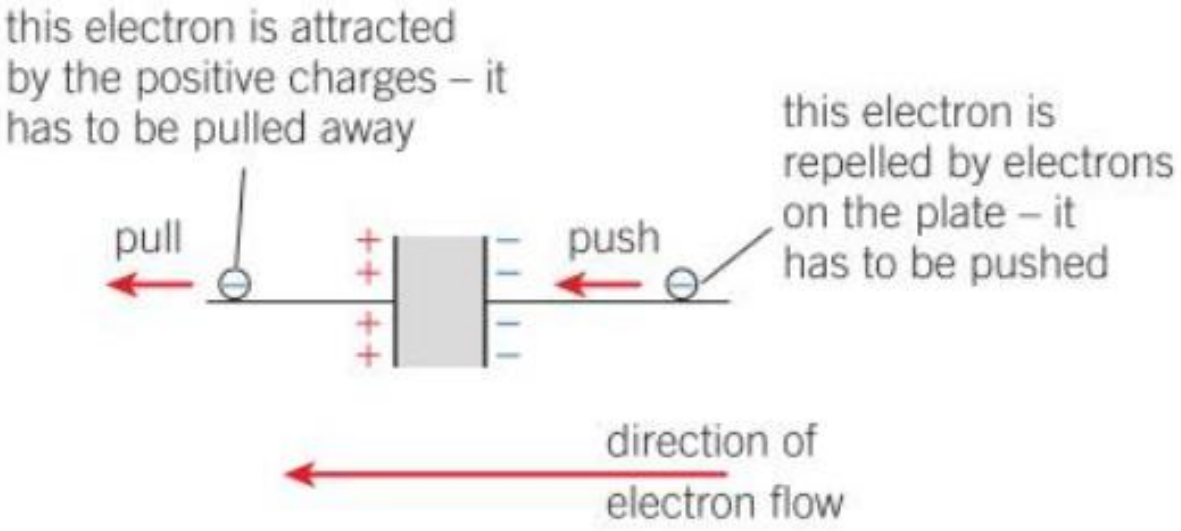
Capacitors as Energy Storage

- Camera flash lamps often use capacitors to store energy ready for use.

- The energy stored depends on the capacitance of and the potential difference across the capacitor.

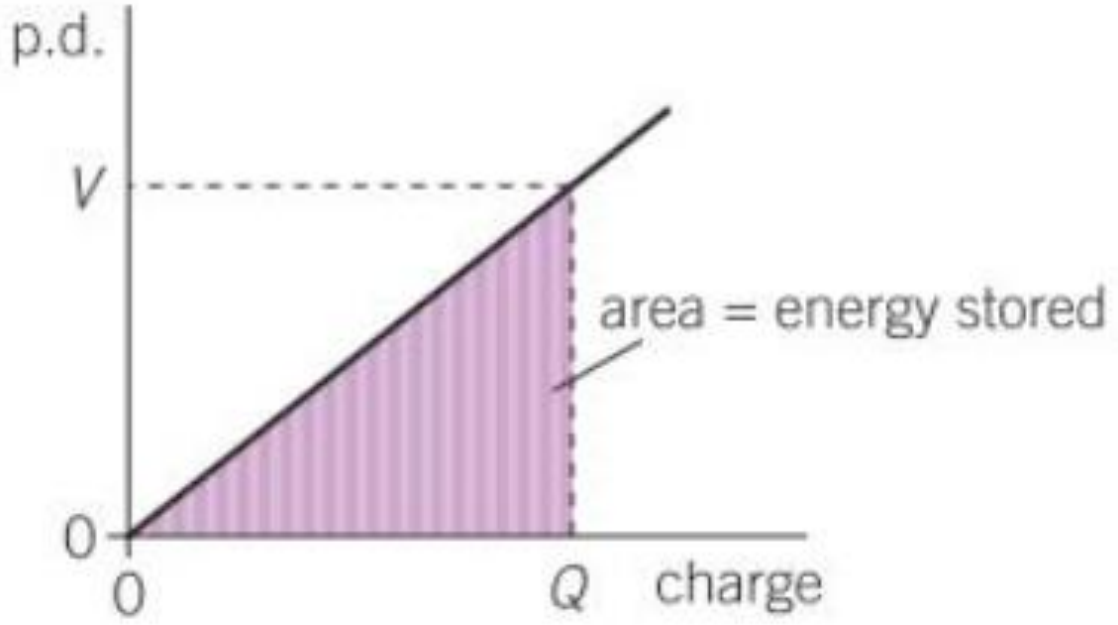
Calculating Energy

- Imagine a capacitor being charged:
  - Each electron moving towards the negative plate will experience a repulsive force from the electrons already on it.
  - Similarly, each electron leaving the positive plate will be attracted back onto the plate.
  - Work needs to be done to add electrons to one plate and remove them from the other.
  - This work is provided by the cell.

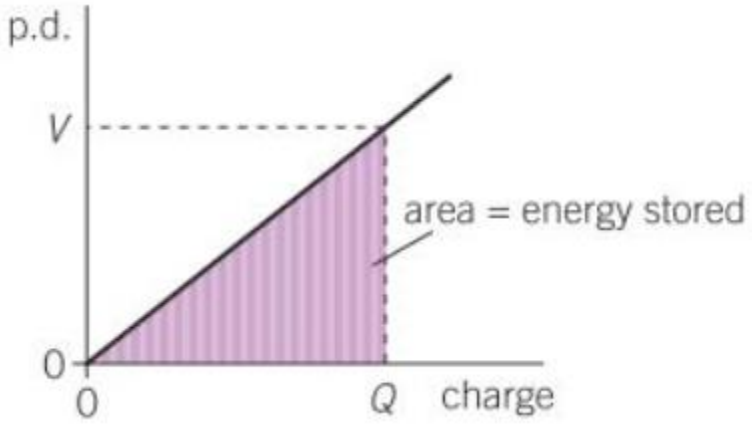


Potential Difference – Charge graphs

- Like elastic potential energy, the work done on electrons is potential, or stored.
- This energy is shown by the area under a PD – Charge graph.



Stored Energy



$$W = \frac{QV}{2}$$

Where:

Work, W, is the shaded area under the graph.

V is the Potential Difference across the capacitor when the charge difference is Q.

More equations

If $W = \frac{QV}{2}$ and $Q = VC$ $V = \frac{Q}{C}$

Then $W = \frac{V^2C}{2}$

So,
$W \propto V^2$ and $W \propto Q^2$

and $W = \frac{Q^2}{2C}$

Doubling Potential Difference or Charge leads to a quadrupling of energy stored.

6.1.2 Energy (review)

6.1.2 Energy

Learning outcomes

Learners should be able to demonstrate and apply their knowledge and understanding of:

(a) p.d.–charge graph for a capacitor; energy stored is area under graph

(b) energy stored by capacitor;

$$W = \frac{1}{2}QV,\ W = \frac{1}{2}\frac{Q^2}{C} \text{ and } W = \frac{1}{2}V^2C$$

(c) uses of capacitors as storage of energy.

6.1.3 Charging & Discharging Capacitors

6.1.3 Charging and discharging capacitors

Learning outcomes

Learners should be able to demonstrate and apply their knowledge and understanding of:

(a) **(i)** charging and discharging capacitor through a resistor

(ii) techniques and procedures to investigate the charge and the discharge of a capacitor using both meters and data-loggers

(b) time constant of a capacitor–resistor circuit; $\tau = CR$

(c) equations of the form $x = x_0 e^{-\frac{t}{CR}}$ and $x = x_0(1 - e^{-\frac{t}{CR}})$ for capacitor–resistor circuits

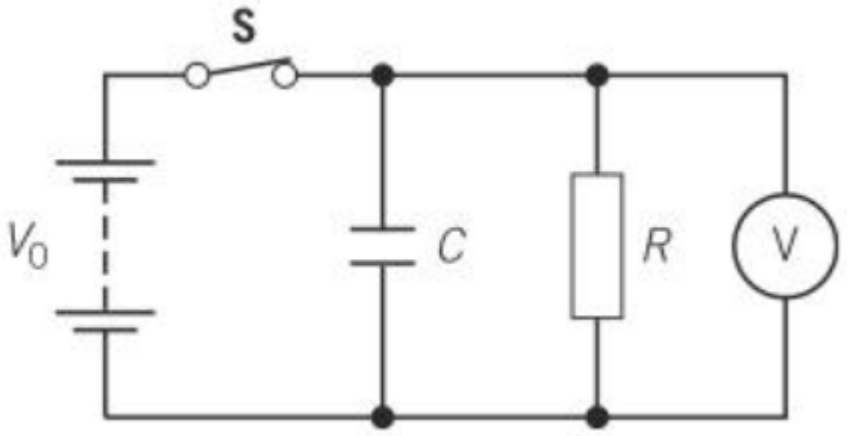
(d) graphical methods and spreadsheet modelling of the equation $\frac{\Delta Q}{\Delta t} = -\frac{Q}{CR}$ for a discharging capacitor

(e) exponential decay graph; constant-ratio property of such a graph.

Capacitor discharge:

Physics event or medical condition?

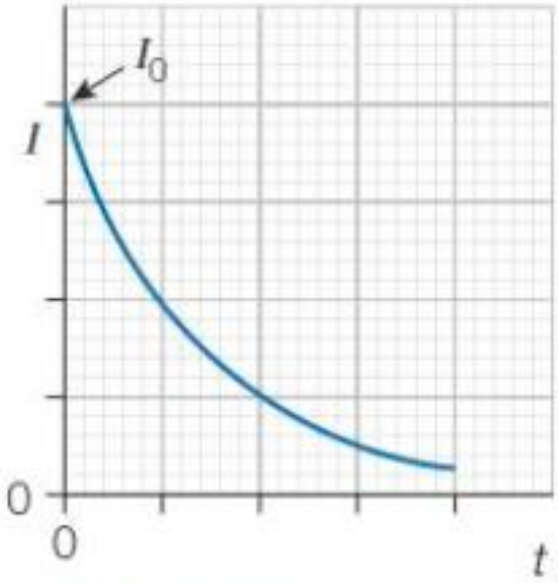
How do capacitors discharge?

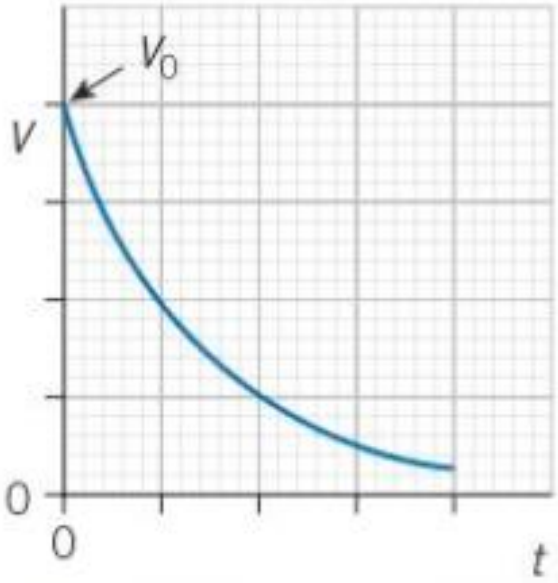


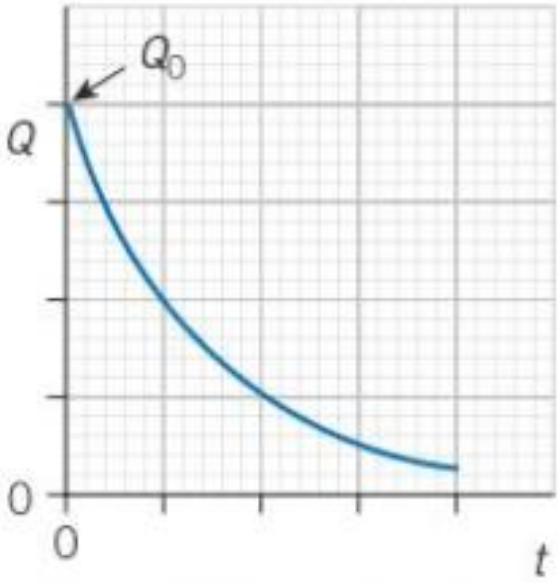
- Close switch, s
  - The capacitor charges. PD across the capacitor is V_0.
- Open the switch and start a stopclock.
  - The capacitor discharges through the resistor.
  - The stored charge decreases over time. (Q = VC)
  - PD across the capacitor decreases over time.
  - Current through the resistor decreases over time. (I = V/R)

- Measure the PD across the capacitor over time.
- Calculate Charge stored in the capacitor & current through the resistor.
- Plot PD, Charge & Current against time & suggest the relationships.

Results







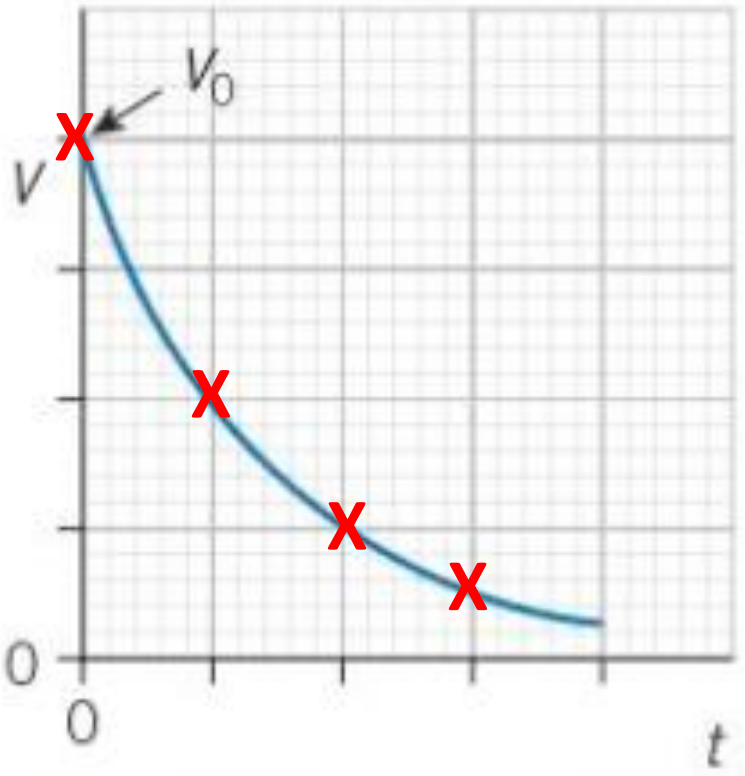
- They all show exponential decay.

$$V = V_0 e^{-\frac{t}{CR}},\ I = I_0 e^{-\frac{t}{CR}},\ \text{and}\ Q = Q_0 e^{-\frac{t}{CR}}$$

Constant Ratio

$$\frac{V_1}{V_0} \approx \frac{V_2}{V_1} \approx \frac{V_3}{V_2}$$

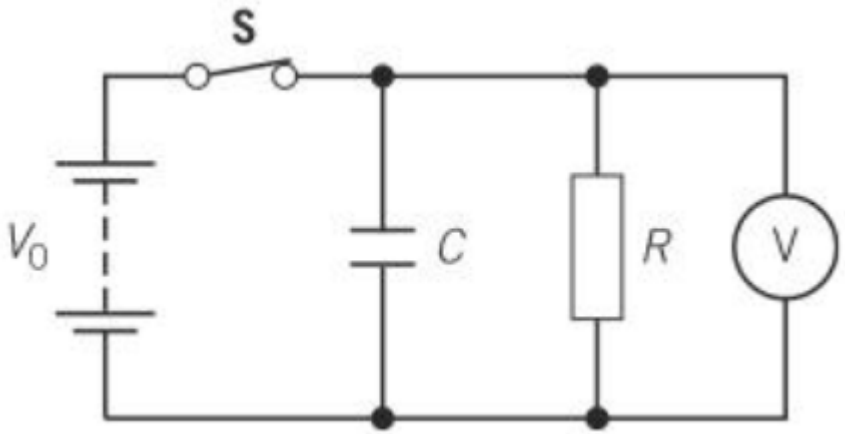
- The discharge curves show a constant ratio of decay over a particular time interval.
  - Select a time interval (say 10s)
  - Calculate the reduction of PD as a ratio at any point on the curve.
  - Repeat for other points



Time Constant, τ=CR

$$V = V_0 e^{-\frac{t}{CR}}$$

- The exponential decay equations (example above) all have a CR term in them.
  - This is the product of Capacitance & Resistance.
  - Multiplying the units farads with ohms gives seconds.
  - This CR term is called the time constant and is a measure of how long the decay will take for a particular circuit.



When t = CR

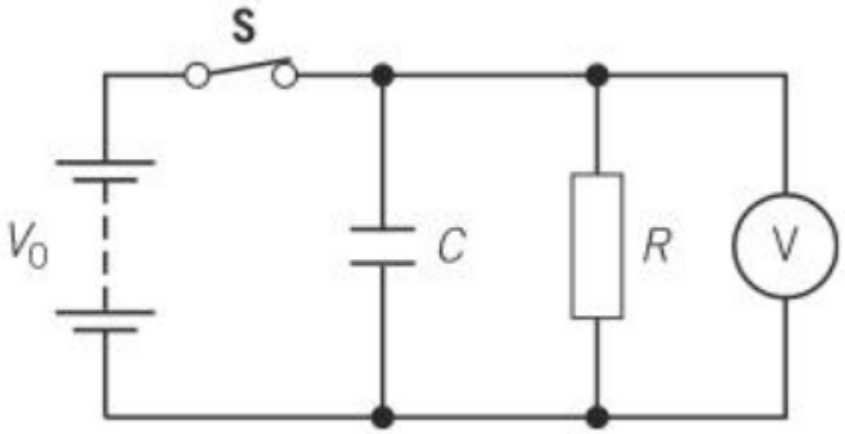
- If we set the time interval, t, to equal the time constant, CR, then:

$$V = V_0 e^{-\frac{t}{CR}}$$

- So, the time constant, τ, is the time taken for the PD (or charge or current) to fall to 37% of its original value.

Modelling exponential decay

- Here the capacitor & resistor have the same PD across them.
- The charge stored by the capacitor is Q=VC and the current is I=V/R



PD is the same for both the capacitor and the resistor so:

$$I = \frac{V}{R} = \frac{Q}{CR}$$

The current as a capacitor discharges can be shown by:

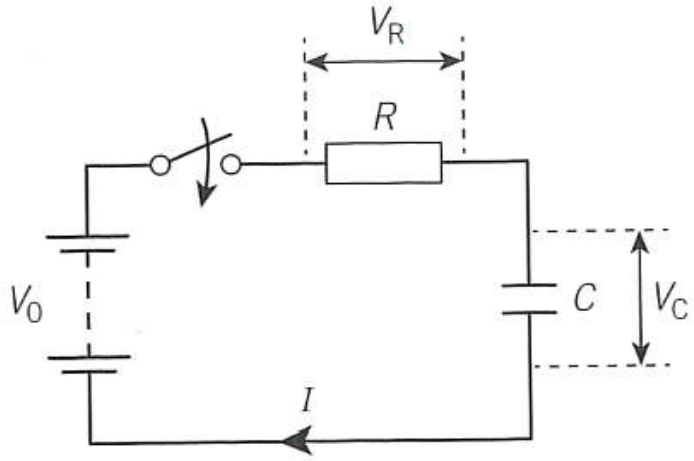
$$I = \frac{\Delta Q}{\Delta t} = -\frac{Q}{CR}$$

Use a spreadsheet to model the decay

- Make a data table
- Show the decay curve graph

So, we've looked at discharge, what about charging?

Charging



- When the switch is closed:
  - Maximum current flows
  - Capacitor begins to charge
  - PD across capacitor (V_C) increases from 0 to V_0
  - PD across resistor decreases from V_R to 0
  - Once charged the current will be zero

Equations

- Current will decrease exponentially as charging progresses:

$$I = I_0 e^{-\frac{t}{CR}}$$

- PD across the resistor will decrease exponentially:

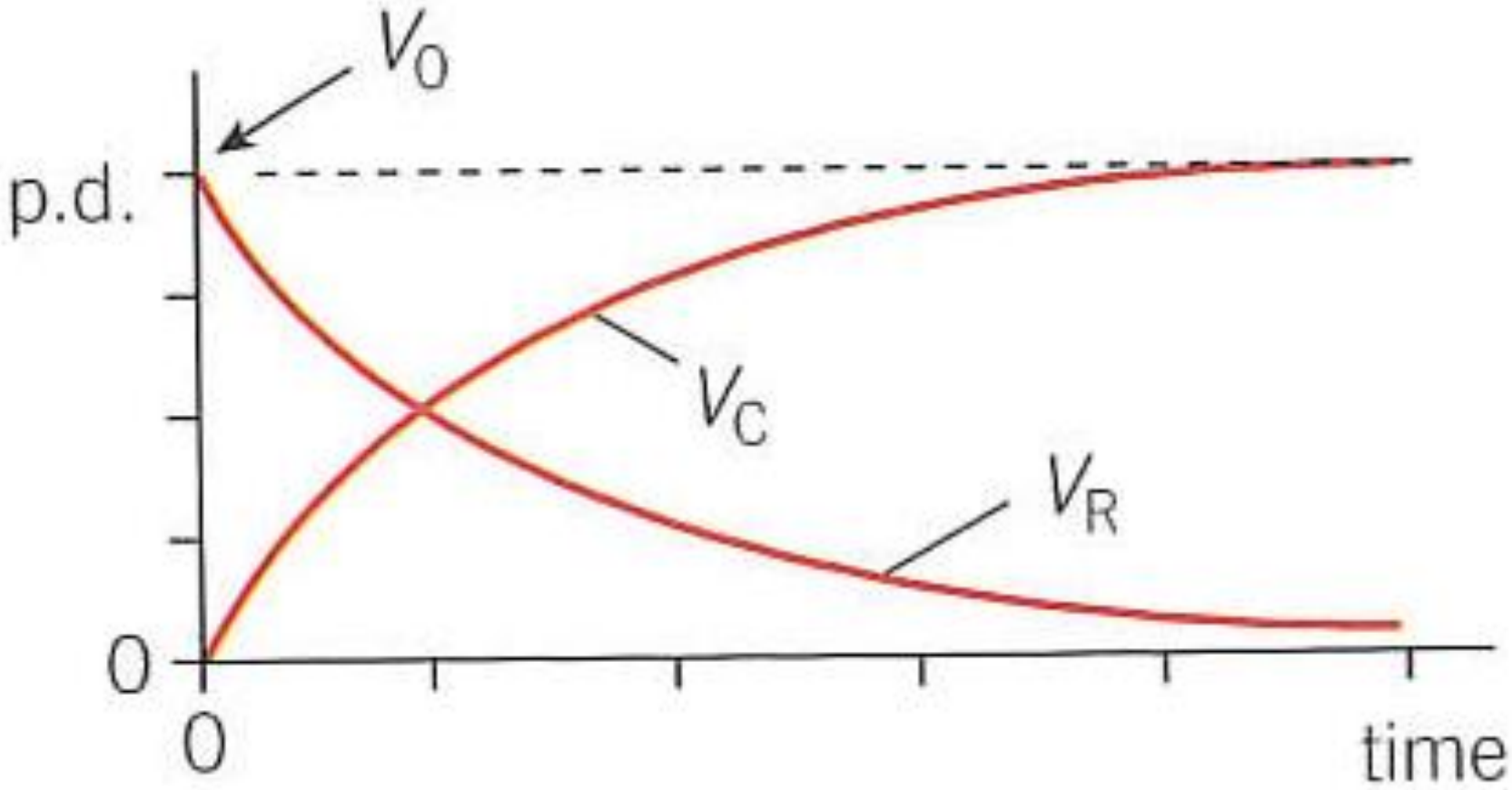
$$V_R = V_0 e^{-\frac{t}{CR}}$$

- PD across the capacitor will increase exponentially:

At any time t, $V_0 = V_R + V_C$, therefore

$$V_C = V_0 - V_0 e^{-\frac{t}{CR}} \text{ or } V_C = V_0 (1 - e^{-\frac{t}{CR}}).$$

$$V_0 = V_R + V_C$$



The Rules

- PD, Current & Resistance are related by V=IR
- PD, Charge & Capacitance are related by Q=VC
- Current, Charge and PD across the capacitor will vary with time according to

$$x = x_0\left(1 - e^{-\frac{t}{CR}}\right)$$

where x can either be Q, I or V

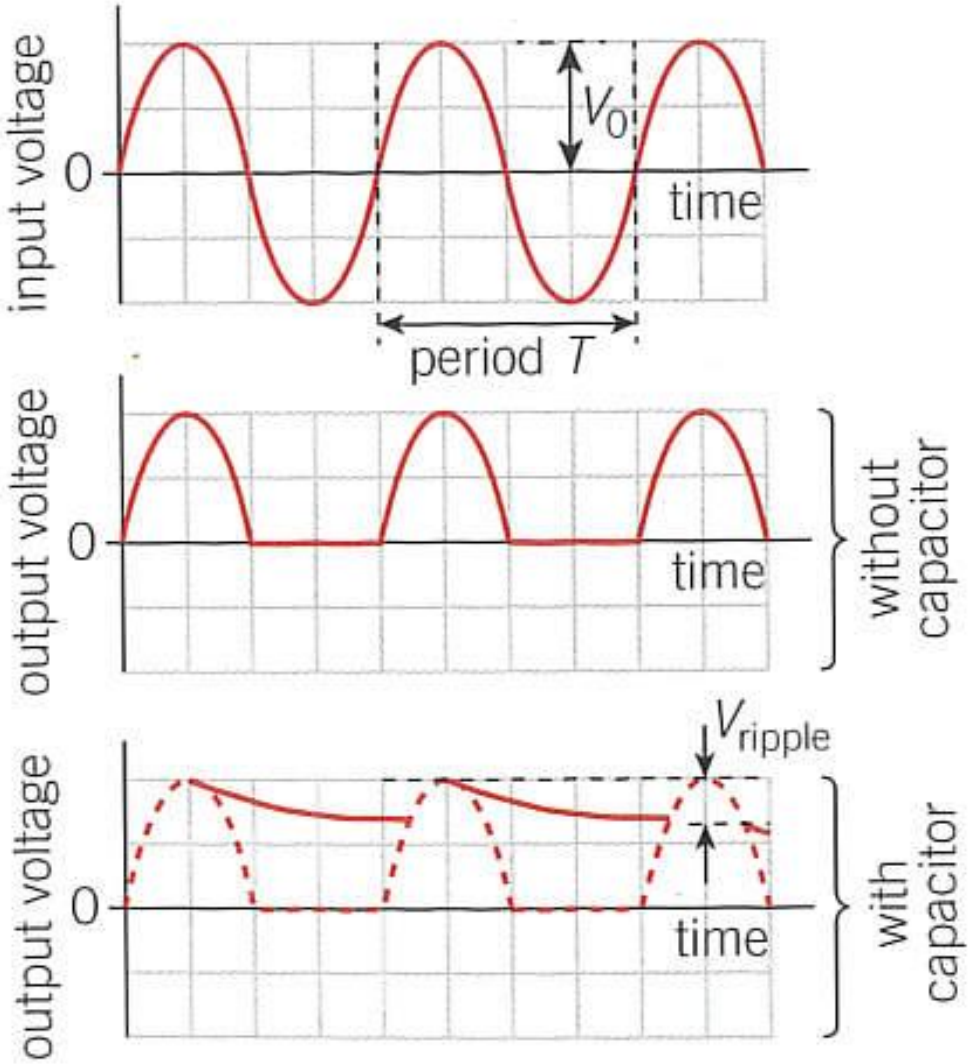
- At any time, the PD across the components adds to V_0 ($V_0 = V_R + V_C$)

Uses of Capacitors

- For high power output:
  - Capacitors cannot store large quantities of charge like a cell but can release it faster to provide high power outputs.
    - Camera flash
- For back up emf source:
  - Capacitors can store energy for use when the main source fails.
    - Emergency supply for a computer
    - Emergency lighting during a power outage
- For smoothing an inverted AC supply:
  - See next…

Smoothing an AC supply

- The input AC supply EMF.
- Diode included to “rectify” the AC supply.
- Capacitor included to smooth the rectified output to a small ripple of almost constant value. Ripple can be reduced with a large time constant.



6.1.3 Charging & Discharging Capacitors (review)

6.1.3 Charging and discharging capacitors

Learning outcomes

Learners should be able to demonstrate and apply their knowledge and understanding of:

(a) **(i)** charging and discharging capacitor through a resistor

(ii) techniques and procedures to investigate the charge and the discharge of a capacitor using both meters and data-loggers

(b) time constant of a capacitor–resistor circuit; $\tau = CR$

(c) equations of the form $x = x_0 e^{-\frac{t}{CR}}$ and $x = x_0(1 - e^{-\frac{t}{CR}})$ for capacitor–resistor circuits

(d) graphical methods and spreadsheet modelling of the equation $\frac{\Delta Q}{\Delta t} = -\frac{Q}{CR}$ for a discharging capacitor

(e) exponential decay graph; constant-ratio property of such a graph.

Module 5 – Newtonian world and astrophysics

5.1 Thermal physics
5.2 Circular motion
5.3 Oscillations
5.4 Gravitational fields
5.5 Astrophysics and cosmology

Module 6 – Particles and medical physics

Complete! →

6.1 Capacitors
6.2 Electric fields
6.3 Electromagnetism
6.4 Nuclear and particle physics
6.5 Medical imaging